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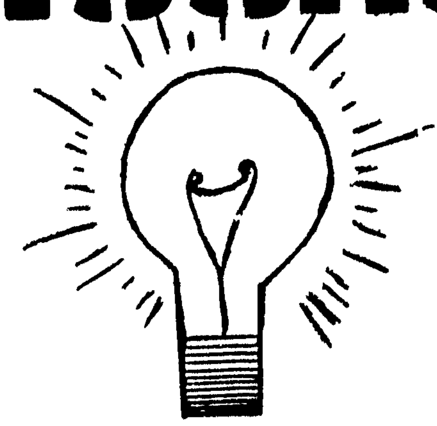
This booklet, one of a series, has been developed for the project, A Program for Mathematically Underdeveloped Pupils. A project team, including inservice teachers, is being used to write and develop the materials for this program. The materials developed in this booklet include (1) number patterns, (2) measuring the interior and exterior angles of polygon , (3) sets and subsets, (4) finding the number of terms and the sum of an arithmetic progression, (5) continued dividing and summing, and (6) counting the faces, vertices, and edges in solids. (RP)

$0+1+2=3$

$1+2+3=6$

$2+3+4=9$

PATTERNS PARTICULARS & GUESSES



$1+3=2\times 2$

$1+3+5=3\times 3$

$1+3+5+7=4\times 4$

$1+3+5+7+9=5\times 5$

ESEA Title III

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PATTERNS, PARTICULARS, AND GUESSES

Number Patterns

An important way of discovering facts in mathematics is finding and examining patterns. A given pattern may be true only for a few specific cases or could be true for every case that "fits" into the pattern. Our interest will be to examine patterns and from some specific examples to offer a general statement.

Consider the sum of three consecutive whole numbers.

Example: $0 + 1 + 2 = 3$

$2 + 3 + 4 = 9$

$8 + 9 + 10 = 27$

From these three "specific" cases, can you make a statement that you think is true in "general?" Examine the sums carefully. A general statement is:

The sum of three consecutive whole numbers is exactly divisible by 3. (It is a multiple of 3.)

Give some specific examples of this general statement--other than the ones used in the example above.

A general statement is given and one specific example of the general statement. Provide three more specific examples of each.

1. The sum of two even numbers is even.

Specific examples: a. $2 + 4 = 6$

b.

c.

d.

2. The product of two odd numbers is cdd.

Specific examples: a. 5 x 3 = 15

b.

c.

d.

3. The sum of any odd number and 2 is also an odd number.

Specific examples: a. 5 + 2 = 7

b.

c.

d.

4. When a perfect square--(1, 4, 9, 16, 25, 36, ...) -- is divided by 3, the remainder is either 0 or 1.

Specific examples: a. $3\overline{)4}$ with quotient 1

3

1 = remainder

b.

c.

d.

5. If a number is multiplied by itself (squared), the product will end in one of the following digits--(0, 1, 4, 5, 6, 9).

Specific examples: a. 6 x 6 = 3<u>6</u>

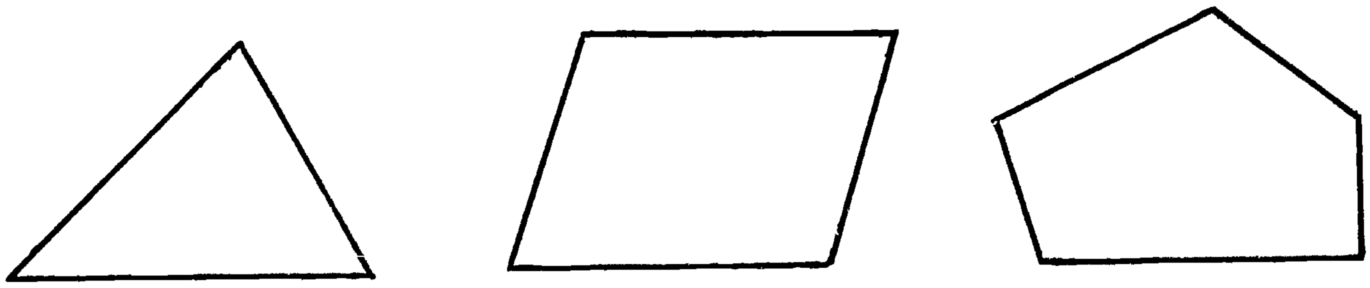
b.

c.

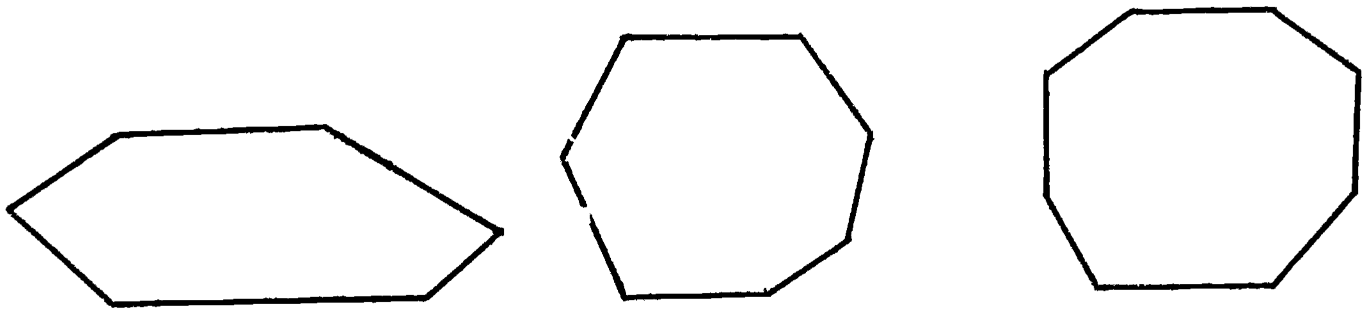
d.

Measure of Interior Angles of Polygons

Consider the problem of finding the sum, in degrees, of the measures of the interior angles of a polygon. Several polygons are pictured below. Can a general statement be made regarding the sum? To start, first determine the total number of degrees in a triangle (the sum of the angles).

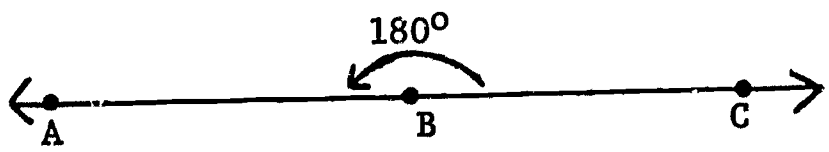


Triangle Quadrilateral Petagon

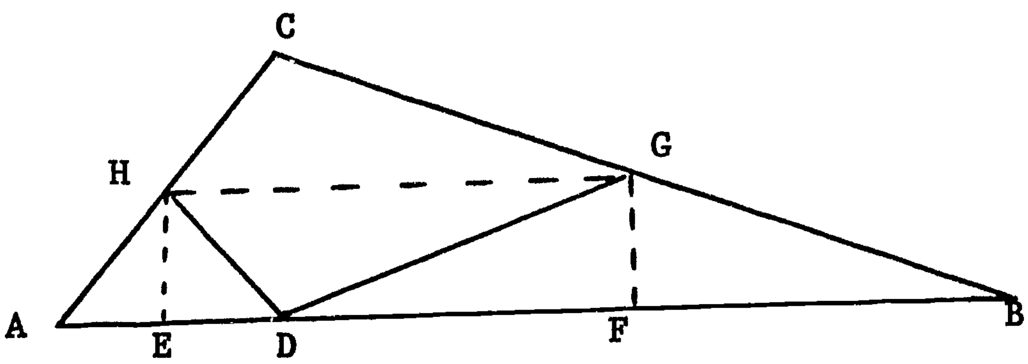


Hexagon Heptagon Octagon

The sum of the measures of the interior angles of a triangle may be determined by recalling that a straight angle has a measure of 180. The angle below is a straight angle and therefore has a measure of 180.

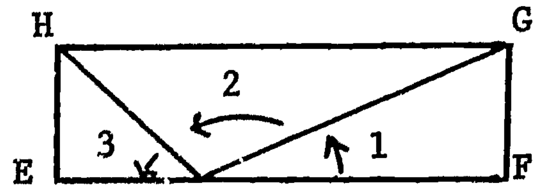


The total number of degrees in the three angles of a triangle may be found in the following way. Using a piece of paper in the shape of a triangle, make the folds as indicated in the drawing. (Fold along the broken line.)



(Fig. 1)

Points A, B, and C will coincide with (fall on) point D, and the completed result will be in the shape of a rectangle EFGH.

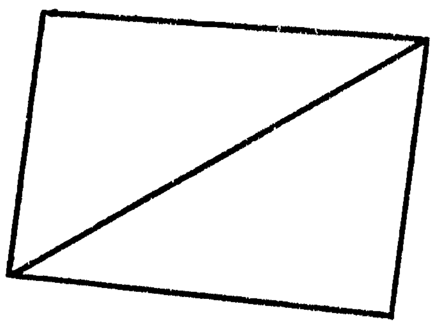


(Fig. 2)

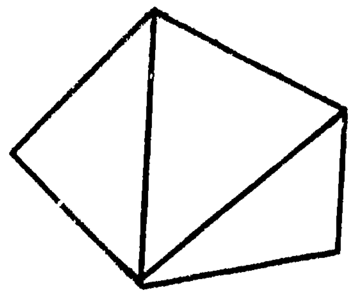
Angle 1, of figure 2, is actually angle B of figure 1. Likewise, angles 2 and 3 are the angles C and A. It can be seen that angles 1, 2, and 3 together form a straight angle, and, therefore, the sum of the degrees of the interior angles of a triangle is 180.

This statement must be agreed upon before the problem of finding the sum of the degrees in the interior angles of a polygon can be solve. It must be accepted that the sum of the measures of the interior angles of a triangle is 180. If you are not convinced of this, it is suggested that you use a protractor and measure the interior angles of several triangles. It may appear to you that the sum in each case is not 180, but this is due to the error involved in measuring.

The number of degrees in the interior angles of any polygon may be determined by separating it into triangles. This is illustrated.



(Fig. 3)



(Fig. 4)

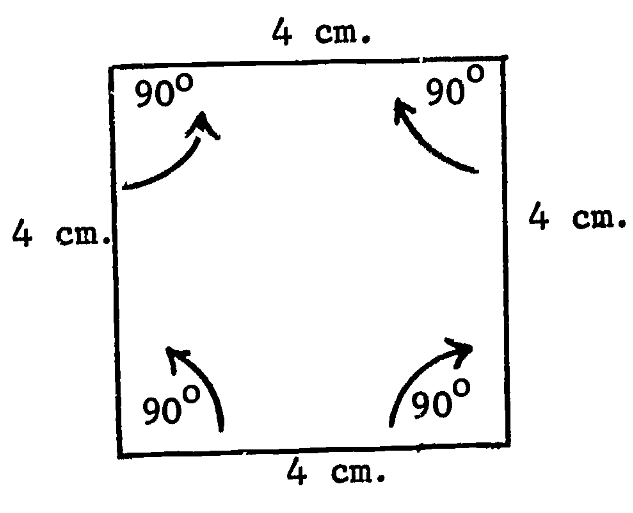
Two triangles are formed in figure 3, and the sum, in degrees, of the interior angles is 2 x 180 , or 360 . What is the sum of the interior angles in figure 4?

Activities

Complete the table below and see if you can determine the number of degrees in the interior angles of any polygon.

NAME OF POLYGON	NUMBER OF SIDES	NUMBER OF TRIANGLES FORMED	SUM OF DEGREES IN INTERIOR ANGLES
Quadrilateral	4	2	$2 \times 180^\circ = 360^\circ$
Pentagon	5	3	________
Hexagon	6	______	________
Heptagon	7	______	________
Octagon	8	______	________
N - gon	N	______	________

A <u>regular</u> polygon is a polygon whose sides all have the same length and whose angles have the same measure. For instance, the quadrilateral which is pictured below is a regular quadrilateral.



Measure of One Interior Angle of Regular Polygons

Activities

How would you determine the number of degrees in one interior angle of a regular polygon? Complete the table below and see if you can answer the previous question.

1.

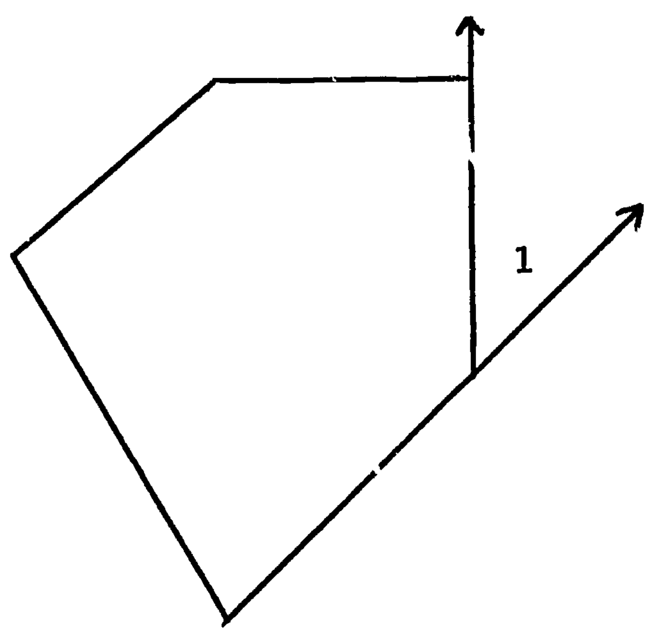
NAME OF POLYGON (regular)	SUM OF DEGREES IN INTERIOR ANGLES	NO. ANGLES	NO. OF DEGREES ONE ANGLE
Square	360°	4	$360^\circ \div 4 = 90^\circ$
Regular Pentagon	540°	5	________
Regular Hexagon	________	____	________
Regular Heptagon	________	____	________
Regular Octagon	________	____	________
Regular N - gon	$(N - 2) \times 180^\circ$	N	________

2. Find the number of degrees in one angle of a regular 20-gon.

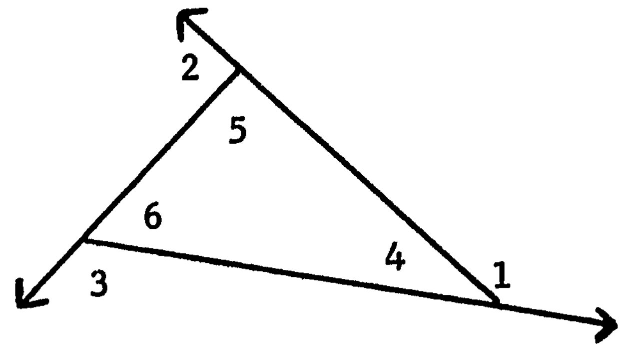
By now it is apparent that the sum of the measures of the interior angles of a polygon with 20 sides is greater than the sum of the measures of the interior angles of a polygon of 19 sides.

Measure of Exterior Angles of Polygons

What about the sum of the measures of the exterior angles, one at each vertex, of a polygon? Angle 1 i~ an exterior angle.



Consider first the exterior angles, one at each vertex, of a triangle.



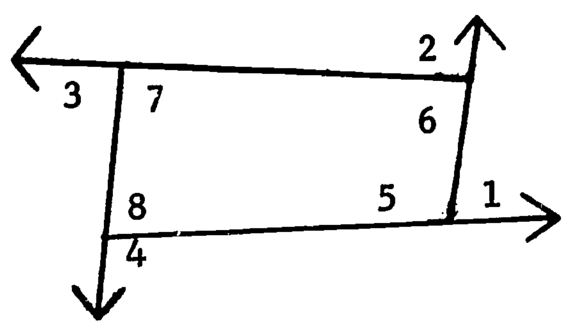
The angle pairs 1 and 4, 2 and 5, 3 and 6 each form a straight angle.

Activities

Find the sum of the measures of each of the following $\angle$s (m$\angle$ means measure of angle in degrees):

1. m $\angle 1$ + m $\angle 4$ = ________
2. m $\angle 2$ + m $\angle 5$ = ________
3. m $\angle 3$ + m $\angle 6$ = ________
4. m $\angle 1$ + m $\angle 2$ + m $\angle 3$ + m $\angle 4$ + m $\angle 5$ + m $\angle 6$ = ________
5. m $\angle 4$ + m $\angle 5$ + m $\angle 6$ = ________; m $\angle 1$ + m $\angle 2$ + m $\angle 3$ = ________

Find the sum of the measures of the exterior angles, one at each vertex, of a quadrilateral.



6. m $\angle 1$ + m $\angle 5$ = ________
7. m $\angle 2$ + m $\angle 6$ = ________
8. m $\angle 3$ + m $\angle 7$ = ________
9. m $\angle 4$ + m $\angle 8$ = ________
10. m $\angle 1$ + m $\angle 2$ + m $\angle 3$ + m $\angle 4$ + m $\angle 5$ + m $\angle 6$ + m $\angle 7$ + m $\angle 8$ = ________
11. m $\angle 5$ + m $\angle 6$ + m $\angle 7$ + m $\angle 8$ = ________
12. m $\angle 1$ + m $\angle 2$ + m $\angle 3$ + m $\angle 4$ = ________
13. What is the sum of the measures of the exterior angles of an octagon? ________ Of a 20-gon? ________

Sets and Subsets

Consider the problem of finding the number of subsets of a given set. First, we need to know what is meant by a set and a subset. The students in this classroom form a set. Also, there is the set of desks in this classroom. A set is usually indicated by braces, {students in this classroom} and {desks in this classroom}. The braces symbolize "the set of," and these two sets are read, "the set of students in this classroom" and "the set of desks in this classroom." A subset of a set is another set consisting of one or more elements of the original set, or the empty set.

Activities

Fill in the blanks below and see if you can find by guessing the number of subsets in any given set.

1.

NO. ELEMENTS IN SET	SET	SUBSETS	NO. OF SUBSETS
1	{a}	{a}, { }	$2 = 2^1$
2	{a, b}	{a},{b}, {a,b} , { }	$2 \times 2 = 2^2$
3	{a, b, c}	{a},{b}, {c}, {a,b}, {a, c},{b,c},{a,b,c} { }	______
4	{a, b, c, d}	____________________	______

2. Without listing the subsets, can you tell how many subsets a set which contains 5 elements would have? __________ A set with 6 elements? __________ A set with n elements? __________

Consider the following subsets of the set of odd whole numbers:

{1, 3}
{1, 3, 5}
{1, 3, 5, 7}
{1, 3, 5, 7, 9}
{1, 3, 5, 7, 9, 11}
{1, 3, 5, 7, 9, 11, 13}

Complete the table by finding the sum of each of these subsets.

(A member of a set is called an element of that set.)

SOME SUBSETS OF THE SET OF ODD WHOLE NOS.	NO. OF ELEMENTS IN EACH SEBSET	SUM	SUM, EXPRESSED AS A PERFECT SQUARE
{1, 3}	2	4	$2 \times 2 = 2^2$
{1, 3, 5}	3	9	$3 \times 3 = 3^2$
{1, 3, 5, 7}	4	___	___
{1, 3, 5, 7, 9}	___	___	___
{1, 3, 5, 7, 9, 11}	___	___	___
{1, 3, 5, 7, 9, 11, 13}	___	___	___

Complete the table by finding the sum of the indicated subset of the set of even whole numbers.

SOME SUBSETS OF THE SET OF EVEN WHOLE NOS.	NO. OF ELEMENTS IN EACH SUBSET	SUM	SUM, EXPRESSED AS A PRODUCT OF 2 INTEGERS
{2, 4}	___	6	2×3
{2, 4, 6}	___	12	3×4
{2, 4, 6, 8}	___	20	___
{2, 4, 6, 8, 10}	___	___	___
{2, 4, 6, 8, 10, 12}	___	___	___
{2, 4, 6, ..., 100}	___	___	___

Arithmetic Progressions

A set of numbers is said to form an Arithmetic Progression, abbreviated A. P., if each number, after the first, is obtained by adding a constant (fixed) value to the preceding one. The set of even whole numbers forms an A. P. The set listed below is an A. P.

$$\{2, 6, 10, 14, 18\}$$

If four is added to the first number, 2, the second number, 6, is obtained.

$$2 + 4 = 6$$

If 4 is added to 6, the result is 10.

$$6 + 4 = 10$$

By adding 4 to any term except the last, the next term is obtained. A set of numbers which does not form an A. P is 4, 7, 11, 12, 15. Why is this not an arithmetic progression?________________________________

Activities

Which of the following sets are arithmetic progressions? (Indicate by writing "yes" or "no.")

1. {1, 3, 5, 7, 9} ____________
2. {0, 4, 8, 12, 16} ___________
3. {9, 9, 12, 16, 21}___________
4. {25, 30, 35, 40, 45}________
5. {3, 3, 3, 3, 3,} ____________

To find a method of calculating the sum of a certain number of terms of an A. P., answer the following questions about the A. P.:

$$\{7, 15, 23, 31, 39, 47\}$$

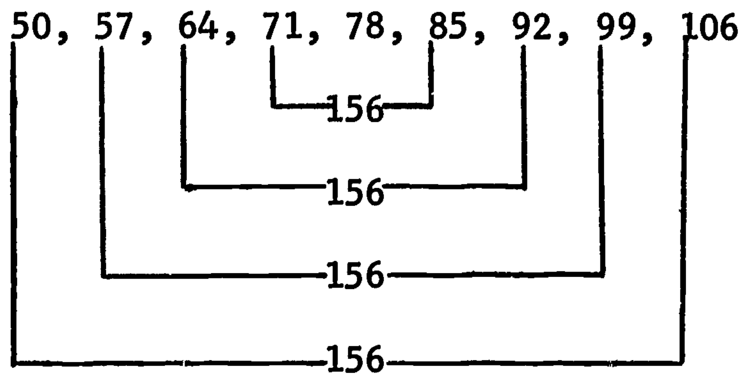
6. What is the sum of the first and last terms? __________
7. What is the sum of the second and the next to last terms? __________
8. What is the sum of the two middle terms? __________
9. What is the sum of the six terms of this arithmetic progression? ________

Activities

Find the sum of each of the following arithmetic progressions:

1. {4, 7, 10, 13, 16, 19, 22, 25, 28, 31}
2. {5, 10, 15, 20, 25, 30, 35, 40, 45, 50 }
3. {3, 8, 13, 18, 23, 28, 33, 38, 43, 48, 53, 58}
4. {27, 33, 39, 45, 51, 57, 63, 69, 75, 81}
5. {103, 111, 119, 127, 135, 143, 151, 159, 167, 175}
6. {50, 57, 64, 71, 78, 85, 92, 99, 106}

In the last problem above there was not a number to be paired with 78. Did you find the solution by pairing 50 with 106, 57, with 99, 64 with 92, 71, with 85, calculating this sum and then adding 78?



The sum is found to be (4 x 156) + 78, or 702. The sum could also have been found by multiplying the middle term by the number of terms. Thus, the sum is seen to be 78 x 9, or 702. Do you see that when an A. P. has an odd number of terms the sum of all the terms may be found by multiplying the middle term by the number of terms?

Activities

Find the sum of each of these arithemetic progressions:

1. {6, 11, 16, 21, 26, 31, 36, 41, 46}
2. {21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61}
3. {16, 26, 36, 46, 56, 66, 76}
4. {101, 103, 105, 107, 109, 111, 113, 115, 117}
5. {13, 20, 27, 34, 41, 48, 55, 62, 69 }

Activities

In each of these arithmetic progressions insert the missing terms or term:

1. 5, 10, 15, _____, _____, _____, 35
2. 8, 11, ____, _____, 20, 23
3. ____, _____, 10, 15, 20, 25
4. 9, 18, 27, 36, 45, ____, _____, _____
5. 4, 11, 18, 25, _____, _____, 46
6. 98, 109, 120, 131, _____, _____, _____, 175
7. 29, 33, 37, _____, _____, 49, _____, _____
8. 32.4, 33.9, _____, _____, 38.4
9. $3\frac{1}{4}$, $3\frac{1}{2}$, $3\frac{3}{4}$, _____, _____, _____
10. $9\frac{1}{2}$, $10\frac{1}{8}$, $10\frac{3}{4}$, _____, _____, _____
11. 5, _____, 15, _____
12. 24, _____, 32, _____
13. 7, _____, 7, _____, _____
14. 195, 190, 185, _____, _____
15. 81, _____, 77, 75, ____, ____
16. 75, _____, 125
17. 235, _____, 265
18. $\frac{3}{16}$, _____, $\frac{9}{16}$
19. 0, _____, 540
20. 357, _____, 422

Activities

Find the missing terms in each of the following. They are not arithmetic progressions:

1. 1, 2, 4, 7, 11, ______, ______
2. 11, 12, 14, 15, 17, 18, ______ , ______
3. 1, 2, 4, 8, 16, ______, ______
4. 1, 3, 9, 27, ______, ______
5. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$,______, ______
6. 0.1, 0.01, 0.001, 0.0001, ______, ______
7. 5, 10, 16, 23, ______, ______
8. 2, 4, 6, 12, 14, 16, 22, ______, ______
9. 4, 5, 6, 8, 10, 13, ______, ______
10. 1, 8, 16, 23, ______, ______

In each of the previous problems which dealt with the sum of an A.P. it was a very simple matter to determine the number of terms in the progression and therefore how many pairs of numbers were in the A.P. A simple process of counting would determine the number of terms in each case. Consider the following set:

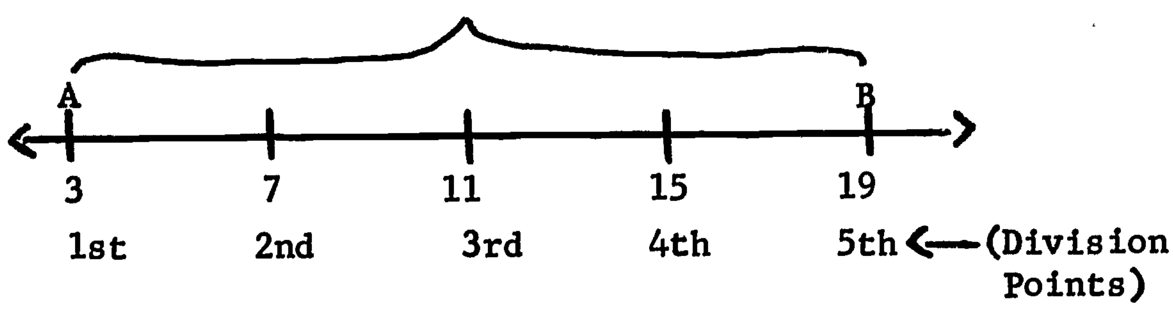
$$\{9, 15, 21, \ldots, 303\}$$

It is possible to write in the missing terms and count them. However, this would be a very long and boring task. Let us find another, shorter way. To find a method or determining the number of terms in an A.P., consider a set which contains only a few terms.

$$\{3, 7, 11, 15, 19\}$$

Graph these numbers on a number line.

(4 segments, each 4 units in length)



What is the length of $\overline{AB}$? ________

Divide $\overline{AB}$ into segments each of which has a length of 4. Now add 1 to this because we are trying to find the number of division points--not just the number of segments of 4 units--which make up the length of $\overline{AB}$

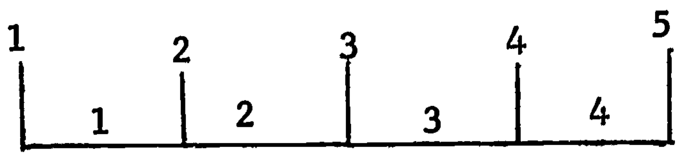
The calculations are given.

$$19 - 3 = 16$$

$$16 \div 4 = 4 \quad \text{(4 segments, each 4 units in length)}$$

$$4 + 1 = 5 \quad \text{(5 division points)}$$

This shows 1 more division point than the number of segments. To illustrate this, picture 5 telephone poles and how many lengths between.



As for the A.P., the number of terms is 5.

Returning to the original problem, 9, 15, 21, ..., 303 it is seen that this set contains ____ elements.

$$303 - 9 = 294$$

$$294 \div 6 = 49$$

$$49 + 1 = 50$$

Summing A.P.'s

What would be the sum of the terms: 9, 15, 21, ..., 303? We find there are 50 elements or terms in this set. The sum is then:

$$\frac{50}{2} \times 312 = 7{,}800$$

If even number of terms: $\frac{50}{2} \times 312$ (the number of pairs times the sum of a pair.)

What would be the sum of the terms: 9, 15, 21, ..., 309? We find there are 51 elements or terms in this set. The sum is then:

$$51 \times \frac{318}{2} = 8{,}109$$

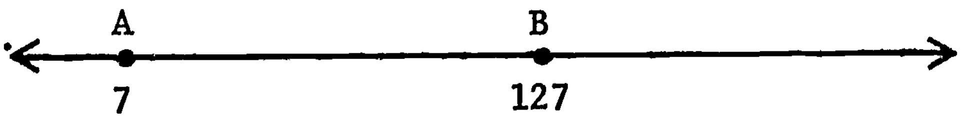
If odd number of terms: $51 \times \frac{318}{2}$ (the number of terms times the middle term which can be found by adding the first and last terms and dividing by two.)

<u>Activities</u>

Find the length of $\overline{AB}$ in each of the following:

1. 

2. 

3. 

4. 

5. 

6. How many segments each 6 units in length can be obtained from $\overline{AB}$ in problem 1? ________ How many division points? ________

7. How many segments each 9 units in length can be obtained from $\overline{AB}$ in problem 2? ________ How many division points? ________

8. How many segments each 4 units in length can be obtained from $\overline{AB}$ in problem 3? ________ How many division points? ________

9. How many segments each 5 units in length can $\overline{AB}$ of problem 4 be divided into? ________ How many division points? ________

10. Give the number of segments of length 2 into which $\overline{AB}$ of problem 5 may be divided? ________ How many division points? ________

In each of the problems 11 - 15 find the sum of the A. P. (Use the results of problems 1 - 10.) The first sum has been found.

11. 9, 15, 21, ..., 303 $\qquad \frac{50}{2} \times 312 = 7{,}800$

12. 32, 41, 50, ..., 167

13. 7, 11, 15, ...,127

14. 17, 22, 27, ...,207

15. 101, 103, 105, ...,187

Continued Dividing and Summing

Take a piece of string 12 inches long. Cut it in half and drop one of the halves on your desk. Take the remaining half and cut it in half. Drop one of these pieces on your desk. What is the total length of the string on your desk now? __________ Again, cut the piece of string in your hand into halves. How much string is on your desk now? _________ At this point there are $6 + 3 + 1\frac{1}{2}$, or $10\frac{1}{2}$, inches of string on your desk. This means that there are $1\frac{1}{2}$ inches of string in your hand, and the halving process becomes somewhat more difficult to perform. After only a few more cuts, it will become physically impossible to continue the halving process. But, let us suppose that this process of halving could be continued for any number of cuts. A table will give an idea of what takes place.

NUMBER OF CUTS	1	2	3	4	5	6
Length of string, in inches, on desk.	6	9	$10\frac{1}{2}$	$11\frac{1}{4}$	$11\frac{5}{8}$	$11\frac{13}{16}$

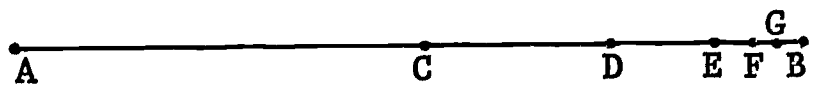
The seventh cutting would place the following amount of string on your desk:

$$6 + 3 + 1\,\frac{16}{32} + \frac{24}{32} + \frac{12}{32} + \frac{6}{32} + \frac{3}{32} = 10\,\frac{61}{32} = 11\,\frac{29}{32}$$

Regardless of the number of cuts, the length of the string on your desk could never be as much as, say, 15 inches. Can you think of a number less than 15 which the length of string on your desk could never exceed?

__________.

This line segment has a length of 2 units. The point C bisects $\overline{AB}$



The point D is the midpoint of $\overline{CB}$, and E is the midpoint of $\overline{DB}$. There will always be a small part of $\overline{AB}$, on the extreme right of $\overline{AB}$, which will have a midpoint. Since AB = 2 units, AC = 1, CD = $\frac{1}{2}$, DE = $\frac{1}{4}$, EF = $\frac{1}{8}$, FG = $\frac{1}{16}$, etc. Form the sum of the lengths of these segments into which $\overline{AB}$ is divided by the "midpoints."

$$\text{(I)} \qquad 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \ldots$$

(Three dots mean that the same pattern is continued on and on.)

Let us analyze several sums, each of which is a portion of (I):

$$1 + \frac{1}{2} = 1\frac{1}{2}$$

$$1 + \frac{1}{2} + \frac{1}{4} = 1 + \frac{2}{4} + \frac{1}{4} = 1\frac{3}{4}$$

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 1 + \frac{4}{8} + \frac{2}{8} + \frac{1}{8} = 1\frac{7}{8}$$

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 1 + \frac{8}{16} + \frac{4}{16} + \frac{2}{16} + \frac{1}{16} = \underline{\qquad\qquad}$$

What is the sum of the portion of (I) which contains 6 numbers? ________

What is the sum of the portion of (I) which contains 7 numbers? ________

What is the smallest number which is "just a little too large" to be the sum of the segments into which AB is divided by midpoints? ________

Could we agree that the sum $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$, where the three dots mean that there is no end to the numbers in the sum, is 2? ________

Will you be able to add enough terms of $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$ to obtain a sum of 2? ________ Could you add enough terms to get a sum of $1\frac{255}{256}$?

Now that this phase of the questioning and answering is complete, let it be said that we can agree that the sum of all the terms of $1, \frac{1}{2}, \frac{1}{4}, \ldots,$ is 2, and this we do. This sum will be called the sum of all the terms of $1, \frac{1}{2}, \frac{1}{4}, \ldots$.

What is the sum of all the terms of $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$?

Solution:

The sum of 2 terms is $\frac{1}{3} + \frac{1}{9} = \frac{3}{9} + \frac{1}{9} = \frac{4}{9}$

The sum of 3 terms is $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} = \frac{9}{27} + \frac{3}{27} + \frac{1}{27} = \frac{13}{27}$

The sum of 4 terms is $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} = \frac{27}{81} + \frac{9}{81} + \frac{3}{81} + \frac{1}{81} = \frac{40}{81}$

The sum of a certain number of terms is always a "little less than" $\frac{1}{2}$.

We can agree that the sum of all the terms of $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$ is $\frac{1}{2}$.

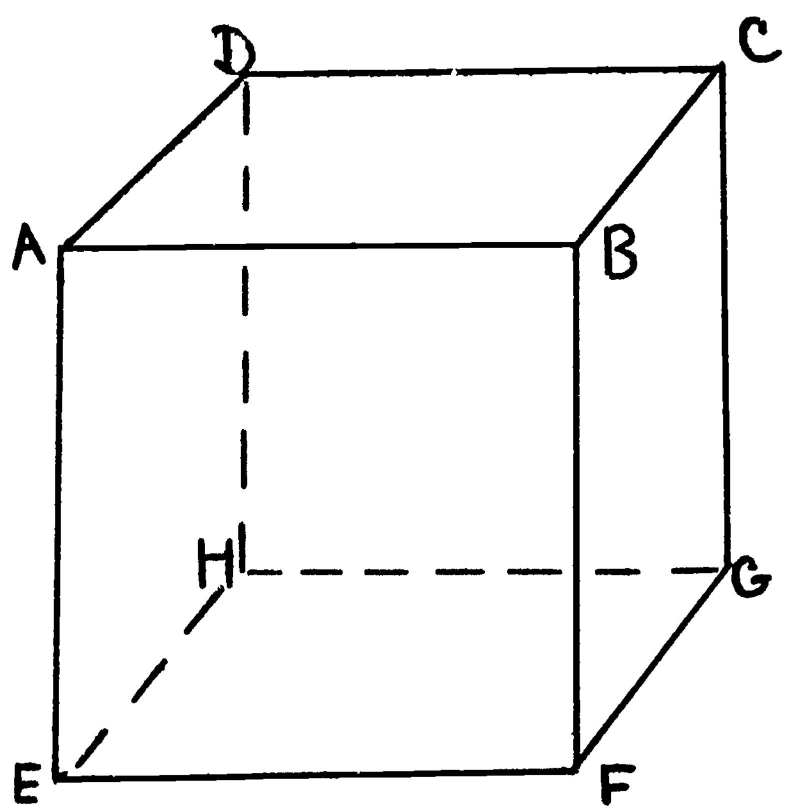
Activities

In each of the following give the sum of all the terms:

1. $4, 2, 1, \frac{1}{2}, \ldots$
2. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$
3. 0.3, 0.03, 0.003, ...
4. 0. 6, 0.06, 0.006, ...
5. $1, \frac{1}{3}, \frac{1}{9}, \ldots$
6. $10, 5, 2\frac{1}{2}$
7. 0.9, 0.99, 0.999,...

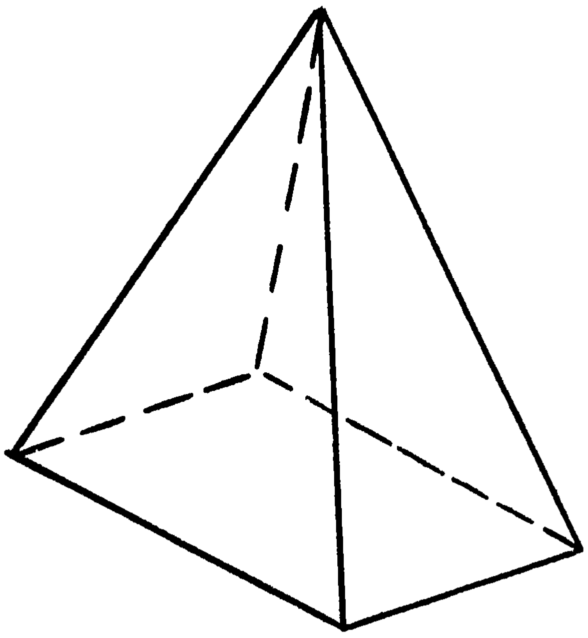
Solids--Faces, Vertices, Edges

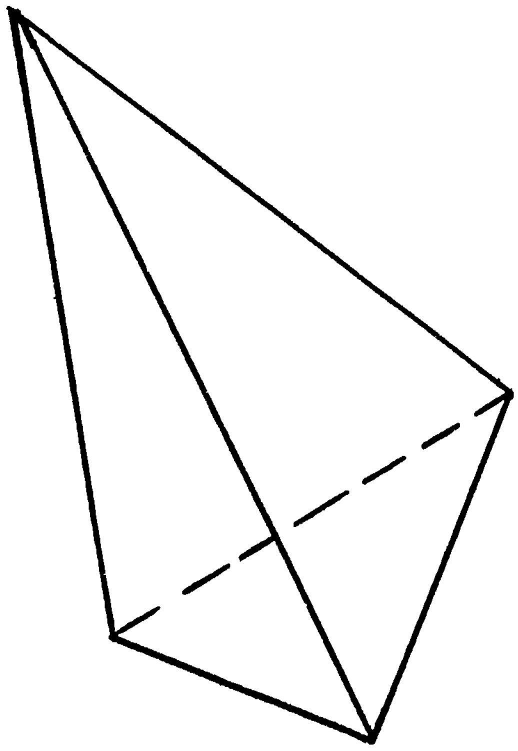
There is a certain relationship which exists among the number of vertices, number of edges, and number of faces of geometric solids. The drawing below will aid in the discussion of this relationship. The rectangles ABCD and BCGF outline two faces of the rectangular solid.

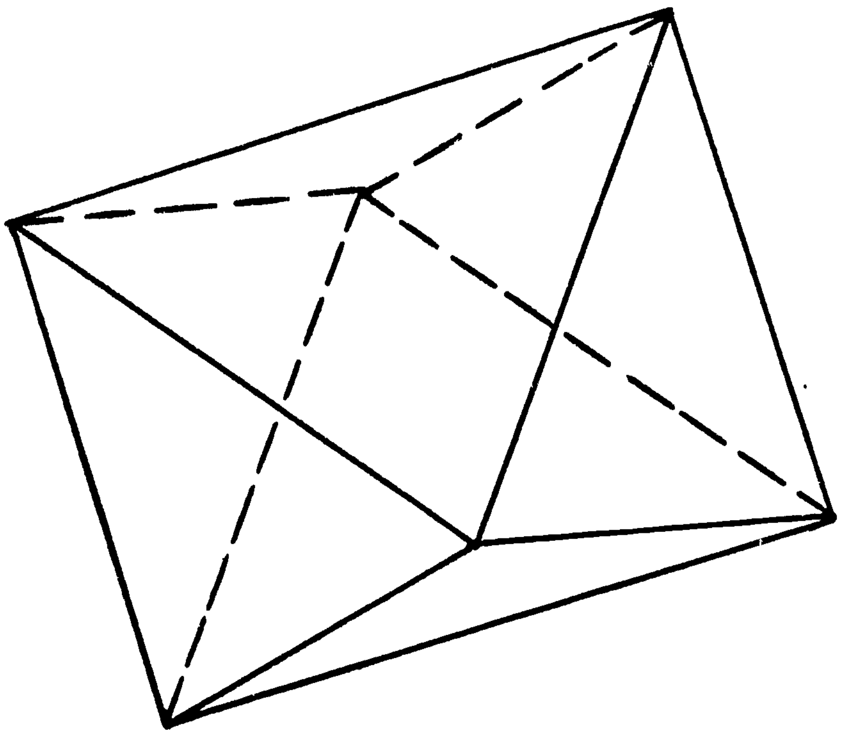


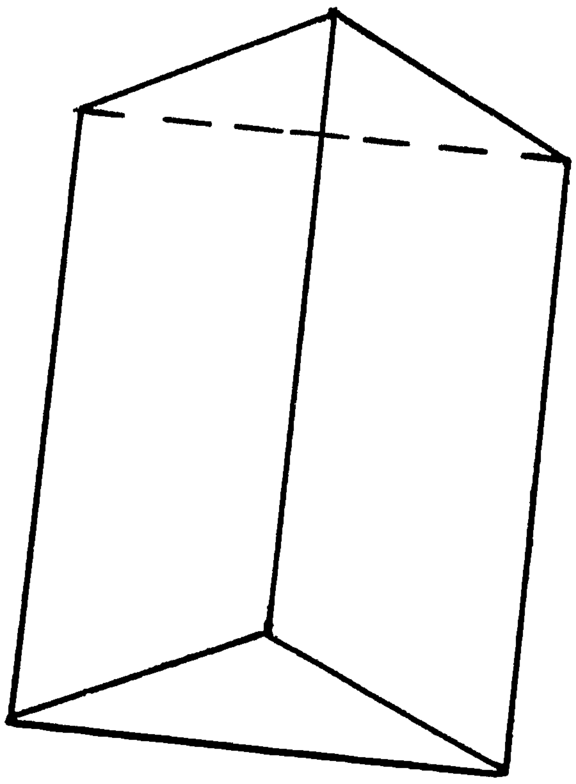
A face of a geometric solid will always be the union of a polygon and its interior. $\overline{AB}$, $\overline{EF}$, or $\overline{AE}$, formed by the meeting of two faces are the edges of a geometric solid. A vertex is the point where three or more edges of a solid meet. Some of the vertices of the rectangular solid above are A, B, and G. Complete the following table, and determine the relationship among the number of vertices, faces and edges of a geometric solid.

NAME OF SOLID	NO. OF FACES	NO. OF VERTICES	NO.OF EDGES
Tetrahedron	4		
Triangular Prism	5		
Octahedron	8		
Hexhedron	6		
Icosahedron	12		
Square Pyramid	5		









PATTERNS, PARTICULARS, AND GUESSES

Illustration of Terms

arithmetic progression, a set of numbers such that each number after the first is obtained by adding a fixed amount to the one before it.

Example: {5, 8, 11, 14}

$$\begin{array}{cccc} & 5 & 8 & 11 \\ & \underline{+3} & \underline{+3} & \underline{+3} \\ 5, & 8, & 11, & 14 \end{array}$$

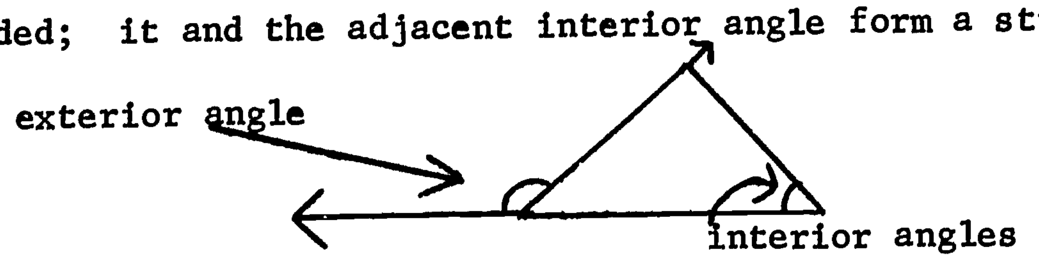
element, any one of the members of a set.

{1, 2, 3}   1 is an element of the set

2 is an element of the set

3 is an element of the set

exterior angles, the angle formed when one of the sides of a polygon is extended; it and the adjacent interior angle form a straight angle.



interior angles, the angle formed where two sides of a polygon meet.

perfect square, a number formed when a whole number is multiplied by itself.

Example:

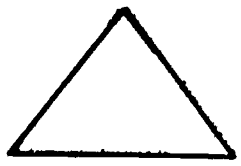
1 x 1 = 1

2 x 2 = 4   { 1, 4, 9, 16, ...} are perfect squares

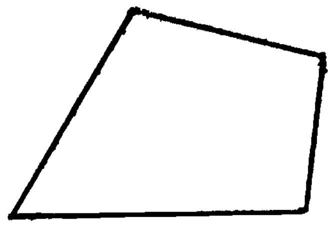
3 x 3 = 9

4 x 4 = 16

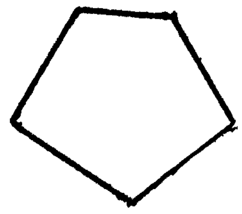
<u>polygon</u>, a plane (flat) figure made of connected straight lines which meet only at their end points.



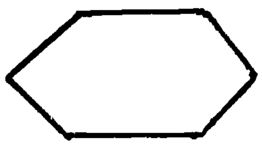
triangle
a 3 sided polygon



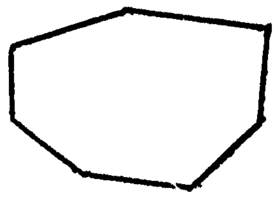
quadrilateral
a 4 sided polygon



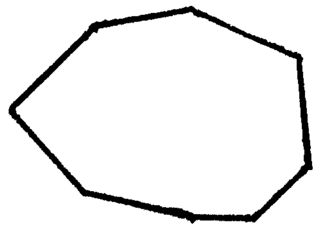
pentagon
a 5 sided polygon



hexagon
a 6 sided polygon



heptagon
a 7 sided polygon



octagon
an 8 sided polygon

<u>regular polygon</u>, a polygon all of whose sides are the same length and all of whose angles have the same measure.

a square is a regular quadrilateral

a stop sign is in the shape of a regular octagon

<u>set</u>, a collection of objects.

<u>straight angle</u>, an angle whose measure is 180.

<u>subset</u>

$A = \{x, y, z,\}$; $B = \{x, z\}$; $c = \{\ \}$

B is a subset of A because the objects in set B all belong to set A. C is another subset of A and is called an empty set.